Book Review

Jody Azzouni. *Tracking Reason: Proof, Consequence, and Truth*. Oxford University Press, Oxford, 2006. vi + 248 pages.

1 Introduction

Jodi Azzouni's *Tracking Reason* offers an interconnected view of truth, mathematical proof, and logical consequence. The overall view is intricate, stimulating, provocative, and insightful, and the writing style is conversational and accessible. In this extended review I will engage Azzouni on two issues: (i) his deflationist approach to truth, and (ii) his syntactic approach to logical consequence. First, however, I will present a short summary of the book; describe what, in my understanding, Azzouni is trying to accomplish in it; and point out a few attractive features of Azzouni's approach, some of which are a bit surprising, given Azzouni's preference for deflationist and empirical philosophy.

2 Azzouni on Truth, Mathematical Proof, and Logical Consequence

2.1 Truth   Azzouni distinguishes two targets of the theory of truth: "*true*" and *truth*. He advocates deflationism with respect to both. Deflationism with respect to truth he regards as a metaphysical position; deflationism with respect to "true"—a linguistic position. The metaphysical claim is that there are no substantive commonalities of truths: there is nothing in common to all truths. The linguistic claim is that the only rule of "true" in discourse is to express "blind endorsement." Thus, if you don't know exactly what Gödel's incompleteness theorem says or what Newton's theory of gravitation says or what Einstein's special relativity says, you cannot assert them directly, but you can still assert them indirectly, or do something that has the same assertoric force as asserting them directly, namely, "blindly endorsing" them. This you accomplish by saying something like "Gödel's incompleteness theorem / Newton's gravitation theory / Einstein's special-relativity theory is true." These complementary claims, namely, that the truth-predicate is no more than a device of blind endorsement and that there is no substantive commonality of truths,

"dethron[e]...the centrality of truth...from our notions of reason and the world" (p. 5).[1] One of Azzouni's goals is to show what "the *real* tools we use to structure our understanding of what there is and how we reason about it" are (ibid.). The two tools he expands on are (i) *anaphorically unrestricted quantification* and (ii) *syntactic derivation*. The latter is discussed in Parts II and III, the former in Part I.

Anaphorically unrestricted quantifiers   Theoretically, the role of "true" as a device of blind endorsement can be explained by means of a regimented, formalized language with anaphoric and quantificational resources that exceed those of natural language. In this language the same variable can occupy both a nominal and a sentential position in the same sentence, and even in the scope of the same quantifier. Its quantification, therefore, can be nominal in one position, sentential in another. In this language we can blindly endorse Gödel's theorem without use of "true" as follows:

(1) $(\forall x)$[Gödel-theorem $(x) \supset x$] or

(2) $(\forall x)$[Gödel-theorem (x) & x].

Azzouni develops a syntax, a semantics, and a proof theory for this language and establishes its completeness. One of its characteristics is the ability to handle blind endorsements of sentences that do not belong to it and are not translatable into it. The existence of this language demonstrates that there is no theoretical need for a truth predicate in our language. It is an accidental feature of natural language that it does not have a sophisticated anaphoric machinery and so has to use an "extraneous" device to execute blind endorsements. Since the only role of "true" is to make such endorsements, and since such endorsements can in principle be made without use of this predicate (or anything requiring substantive truth conditions), the deflationist approach to "true" is justified. Put otherwise, a deflationist truth predicate is sufficient for blind endorsement; hence it is sufficient for our language.

As for *truth*, Azzouni says that the only way for truth to be substantive is for there to be substantive commonalities among truths, and further, that such commonalities must involve truthmakers. But as a nominalist (of the kind described in Azzouni [1]) he claims that while some truths (e.g., physical truths) are made true by truthmakers, others (e.g., mathematical truths) are not. He concludes that there is *nothing* (substantive) common to *all* truths, and therefore a deflationist notion of truth is sufficient.

Another aspect of truth that Azzouni expounds is the paradoxes of truth. I will describe his treatment of natural language in connection with the paradoxes in Section 3 below.

2.2 Mathematical proof   Azzouni offers an account of human reasoning at its best—mathematical proof. Mathematical proofs are for the most part carried out in the vernacular, but their force, Azzouni argues, comes from a systematic correlation with syntactic derivations in some axiomatic system. Azzouni calls this view "the derivation-indicator view of ordinary mathematical proof." Here is how he explains it:

> [The derivation-indicator view of mathematical practice is, briefly] the view that a mathematical proof of *B* from *A* *indicates* that there is a mechanically recognizable derivation from (a proxy of) *A* to (a proxy of) *B* in an algorithmic system.

> The claim isn't that mathematicians recognize that this is what an ordinary mathematical proof does.... Rather, it's that the practice of theorem-proving

in mathematics was shown (empirically) to be of this form by the success of regimenting mathematical proof in algorithmic systems based on first-order logic. Mathematicians recognize...the success of a mathematical proof by what (phenomenally) feels like a grip on a *semantic* relation—specifically implication—between the steps in that proof: a semantic relation that intuitively feels based upon what the statements in the proof say (what they're *about*). Here—as so often—phenomenal feels are utterly misleading. (p. 119)

[W]hat makes proofs convincing are factors that lead to those proofs being correlative to derivations...[where] derivations are (in principle) mechanically checkable, and...the algorithmic systems that codify which rules may be applied to produce derivations in a given system are (implicitly or...explicitly) recognized by mathematicians. (p. 143, changed paragraph order)

The following points provide an outline of the "derivation-indicator" view:

A   Mathematicians construct proofs in languages that are for the most part natural languages, but natural language is not designed for the construction of proofs. In constructing proofs in natural language we seem to be talking about abstract objects and their properties. A proof seems to say that given that certain (abstract) objects have certain properties (stand in certain relations), other objects have these properties (or these objects have other properties). But this is not the real force of a mathematical proof.

B   Mathematical proofs are made correct (convincing, universally agreed upon) through being correlated with certain (valid) derivations within one or another mechanical, effective, algorithmic system. This correlation is of the same kind as Church's correlation between (intuitively) effective and recursive procedures (Church's Thesis). The derivations themselves need not have an ontological status; what is important is that proofs are correlated with something that a "good" algorithmic theory says is a valid derivation. The mathematician usually has no access to these derivations, and therefore he thinks his proofs are valid due to their claiming something true about existing abstracta based on something else which is true about existing abstracta. But this is an illusion. It is important to note that there are no fixed derivations that mathematical proofs are correlated with; instead there is an open-ended family of derivations they are correlated with (sometimes ambiguously) and between which they may shift.

C   The exact relation between proofs and the derivations they indicate, and how it convinces the working mathematician, and the community of mathematicians, that a given proof is correct, seems hard to pin down. Azzouni tries to describe it in various ways. Sometimes it is not clear whether his descriptions are coherent. For example, in one place he declares,

One thing I *do* claim is that the mathematician *does* sense that any traditional proof can be expanded so that "no steps are missing." (p. 122)

Yet immediately he adds,

The mathematician engaged in traditional theorem proving [is] unaware of this, and one reason for his unawareness is that the phenomenological sense of a guarantee...isn't due to one's feel for the shape of a mechanical derivation that can be correlated to a traditional proof. (ibid.)

The closest things I found to something that is really informative are these:

No ordinary mathematical proof indicates a derivation in the sense that it (or a designating term contained within it) *refers* to such a thing or describes its properties. Rather...an ordinary mathematical proof is a combination (often a complex one) of having a particular form, coupled with explicit allusions to suppressed details, that *shows* that a derivation (of such and such a sort) exists. (p. 158)

[B]y what mechanism *mathematicians* provided proofs that indicated derivations without having any clue that this was what they were doing[?] The key to this epistemic puzzle turns on the use of tacit assumptions. It's notorious that mathematicians often...rely on tacit assumptions only teased out by later practitioners.... The recognition, on the part of the profession, that the assumptions are tacit, shows recognition that the proofs in question are incompletely analyzed.... (pp. 159–60)

A standard mathematical proof *indicates* any of a family of derivations without those derivations (i) being what standard proofs abbreviate, (ii) being, in some more extended sense, the "logical forms" of such proofs, or (iii) being items that such proofs are "reducible to." Instead, ordinary mathematical proof, by (among other things) unsystematic combinations of genuine derivation sketches, allusions to such sketches elsewhere in the literature, and metaderivational considerations, convinces mathematicians that the proof is valid because (although the mathematician needn't know this) the proof corresponds to a derivation of such and such a sort. (p. 173)

D   The account is "formalist" in some respects but differs from traditional formalism in others: on the one hand, it emphasizes the connection between proofs and syntactic (effective, mechanical) derivations; on the other, it denies that "derivations can replace proofs altogether in mathematical practice" (p. 141).

2.3 Semantics and logical consequence   Azzouni claims that the semantic notion of logical consequence is useless and misleading:

[W]hat has long been taken to be a straightforward intuitive notion of semantic entailment[,] *if A is true, then B must be true*, is a notion that doesn't actually illuminate our grasp of what follows from what, but largely encapsulates our ignorance of how inference operates. (p. 117)

Instead Azzouni argues that it is "the syntactic motors of inference" (p. 224) that are responsible for ordinary consequences. It is only the existence of a first-order syntactic deduction connecting A to B that explains why A implies B. The reason we think of consequence in semantic terms to begin with is (i) "[o]rdinary language isn't an ideal device for communication"; it is a "gerrymandered product...of evolution" (p. 230), and (ii) we have no choice but to reason in ordinary language, and we are unable to reach the real mechanism underlying implication by introspection. This, however, is "entirely compatible with a...result that our practice is nevertheless in thrall to the indication of mechanically recognizable derivations" (p. 206).

   The process leading to the formation of a semantic conception of logical consequence is psychologically as follows:

[I]ntrospectively, when we reason, we don't feel as if we're manipulating syntactic rules. Instead, we seem to maneuver the steps...by a failure to see how it is possible for the later step to be false if the previous one is true. (p. 226)

This is the source of Tarski's semantics. Our inability to see how it is possible that *A* is true while *B* is false leads us to think in modal as well as alethic terms:

A implies B iff it is impossible that A is true and B is false.

And this in turn is translated into a metaphysical intuition that says there is no possible world in which *A* is true and *B* is false. The latter is represented by Tarski in model-theoretic terms:

A implies B iff there is no model in which A is true and B is false.

But this definition, Azzouni argues, cannot be right. The reasons include those adduced by Etchemendy [2] for the claim that Tarski's definition fails to capture the modality involved in the intuitive concept of logical consequence, plus the following: (i) Tarski's definition rests on an undiscerning intuitive notion: "psychologically[,] the impression [of a guarantee that an intuitive consequence] gives us is as forceful when we're reasoning correctly as when we've made a mistake" (p. 226), and (ii) Tarski's definition yields an implication relation, and "implication relations, in general, are quite unusable: On the model theoretic view... it's impossible to see how anyone can determine directly (i.e., by checking each such model) that in every model that *A* is true in, *B* is true as well" (p. 122).

This does not mean that we have to reject the semantic notion of logical consequence altogether. This we cannot do. But we can show (i) that "the uses of *truth* and *modality* that appear in [the ordinary/semantic notion of validity] can be construed in accord with the deflationist view of truth," and (ii) that "the 'syntactic' view of mathematical proof... is compatible with that ordinary semantic notion—as it's given to us phenomenally when we recognize that a proof is valid" (p. 192). And this we can (presumably) do based on the analysis of truth and proof in the first two parts of the book.

Going in the opposite direction Azzouni declares,

> The test case, as it were, both of the sorts of truth deflationism I'm committed to and the derivation-indicator view of mathematical proof, arises... in their compatibility with the notion of ordinary consequence. (p. 193)

In this way the three parts of *Tracking Reason* are tied together.

3 Attractive Methodological Features

Although Azzouni emphasizes the empirical nature of his work, what he does in this book is not just describing some empirical facts, or establishing such facts empirically. Rather, the book offers a rich theoretical outlook on its three subject-matters— truth, mathematical proof, and logical consequence.

While I find myself at a loss to evaluate Azzouni's claims on the empirical level (on that level the book is largely speculative, since virtually no systematic empirical evidence is offered), on a more theoretical level his claims are easier to evaluate. Four attractive elements of Azzouni's theoretical methodology are (i) functional analysis, (ii) abstract constructions, (iii) dynamic outlook, and (iv) creative contributions.

3.1 Functional analysis   Azzouni's arguments for a deflationist approach to truth are for the most part functional. For example, with respect to the truth predicate, his arguments focus on the following questions: What is (are) the role(s) of "true" in the vernacular? Can this (these) role(s) be fulfilled, theoretically, by other devices? Does any of these roles require a substantive notion of truth? What is the function

of transliterating truth statements in formal language? Is blind endorsement the only role of "true"? What is the role of "true" in truth-conditions clauses? And so on. Similar questions are asked with respect to other concepts: What are regimentations needed for? What is their purpose? And so on.

3.2 Abstract constructions   In approaching his tasks Azzouni's main method is abstract and theoretical: regimentation. What is regimentation and what does it accomplish?

> A regimentation, as I understand it, of a designated section of ordinary language *replaces* that designated section with a (piece of) engineered artificial language, not in the sense of giving speakers a different language to (as a practical matter) use, or speak in, but more narrowly, of giving *normative constraints* on inferences, and other logical matters. (p. 75)

Regimentation brings to light what natural language, due to its evolutionary nature, hides:

> [Natural language is an evolutionary product, and] evolutionary structures are jerry-built on structures already in place for a sequence of quite different ...reasons [from those associated with their current roles]. Thus linguistic devices from the vernacular often have properties that are, given their current functional roles, irrelevant.... Despite the subtlety of evolved structures, there is always the likelihood that aspects of it are useless or even pernicious (given its current niche), and that something else, designed from scratch, could do the job much *better*.... [T]his is why *regimentation* (in something like the Quinean sense) is... of interest. (pp. 74–75, footnote embedded)

The purpose of regimentation, or the building of theoretical linguistic systems is

> (i) to systematically present sentential vehicles with computationally transparent and tractable inferential properties, and to supply a mathematically tractable semantic theory for such items...; and (ii) to use results about such systematically presented sentential vehicles... as the final court of appeal regarding logical issues about the ordinary-language statements the regimentation concerns. (p. 75)

The philosophical tasks for which Azzouni uses, or appeals to, regimentations, or theoretical system building, include (a) showing what is and what is not involved in blind endorsement, (b) showing how natural language can be coherently used in spite of its inconsistency, (c) showing what the norms that sanction mathematical proofs are, and (d) showing what the norms grounding semantic implications are. Thus, by constructing a system for blind endorsement that does not involve a truth predicate at all, Azzouni sets out to show (i) that no substantive truth predicate is needed for such endorsement and (ii) that what is really at work in such endorsement are certain anaphoric and quantificational tools. By appealing to *consistent* artificial languages in which most worthwhile linguistic functions are preserved he shows that natural language is essentially coherent in spite of its inconsistency and that the semantic paradoxes are essentially an evolutionary accident. By systematically correlating successful mathematical proofs with formal derivations, Azzouni claims to unravel the norms sanctioning them and the source of their universal acceptance. And by constructing "complete" systems of logical consequence (in the sense of Gödel completeness), Azzouni argues, we can show that there is nothing more to the semantic notion of logical consequence than is captured by the syntactic-derivational notion.

3.3 A dynamic outlook   Azzouni conceives linguistic practices as dynamic: we can always introduce new resources into language and remove existent ones, and as a result our linguistic practices are not limited by the actual state of our language. For example, we are not limited to endorsing sentences expressible by our language in its current state:

> We…blindly truth-endorse (and blindly false-endorse) sentences howso-ever we need to, and regardless of whether our own language—and its *T*-biconditionals—successfully enables that endorsement: We talk as if every sentence (or proposition) whatsoever falls under our truth predicate or under its anti-extension.…[W]e treat the reach of our truth predicate as the whole field of what is—in principle—truth-apt, even though our actual resources fall woefully short. (p. 112)

We do so since we know that we can "forge access" to sentences outside our language by "enriching the vocabulary of our own language." We can refine "the list of *T*-biconditionals: winnowing items that breed contradiction (or ignoring them) and including new *T*-biconditionals for new vocabulary" (ibid.).

Similarly, in the case of mathematical proof,

> the derivation-indicator view never traps the mathematician within a single algorithmic system. The mathematician shifts from algorithmic systems to stronger ones almost unconsciously. (p. 121)

3.4 Creative contributions   The three methodological principles listed above—functional analysis, theoretical regimentation, and a dynamic approach to language—are exemplified by Azzouni's creative account of blind-endorsement, a variant of the prosentential account of truth. In constructing an artificial language whose task is to identify a certain function of our language, Azzouni argues, we are not bound by (many of) the rules governing this language. For example, we can use pronouns, or variables, that unlike the pronouns of our language play *two* distinct roles rather than one, and play them not just in the same sentence, but in the scope of the same quantifier. Thus, to explain the function of blind endorsement—to explain what is really going on when we use the truth predicate to blindly endorse a sentence—we invent pronouns that play two such roles and quantifiers that bind them in both roles:

> We can (artificially) introduce prosentences, or more precisely, impose an ad-ditional prosentential capacity on a pronoun already in English ("it") so that it can now also appear in sentential positions but still refer back to quanti-fiers (in English) that—when functioning as they ordinarily do—only accept anaphora from pronouns in nominal positions.…I'll call English, absent the truth predicate, but supplemented with anaphorically unrestricted pronouns, *Anaphorish*. (p. 23)

The main idea is that the same linguistic unit can play two roles: the role of a term and the role of a sentence. Furthermore, the unit, in its term role, can serve as a canonical name of itself in the sentence role. Using this method, Azzouni identifies the mechanism at work in a sentence like

(3)   Some sentence is true although no one can prove it

by regimenting it as

(4)   Some sentence, it, although no one can prove it (p. 23)

or as

(5) $(\exists x)$[sentence x & x & $(\forall y)$(Person $y \supset \sim y$ can prove x)].

The variable "*x*" is nominal in its first occurrence, sentential in its second, and nominal again in its third. And the quantifier binding it in all three occurrences is *anaphorically unrestricted*. This innovative theoretical quantifier enables us to "navigate between use and mention" (p. 59), while ordinary-language quantifiers do not.

One distinctive characteristic of anaphorically unrestricted quantification is that it does not require the existence of tokens of blindly-endorsed sentences. Consider the sentence "What John says is true":

> I say: "What John says is true" or "There is something such that *it* is what John says and *it*." What *it* is that? The sentence(-on-a-sense, perhaps),...*a token of which was uttered by John*.   AU[anaphorically-unrestricted]-quantification—to give the right answers for the truth values of its blind endorsements—doesn't require that any token (tracking the content of John's utterance) successfully *substitute* for "it" in my utterance; more remarkably, it doesn't even require that the statement expressed by John's utterance have more than one token. What it needs is only that the variable in sentential position successfully carry the "force" of assenting to the utterance the variable refers to. (pp. 70–71)

In this way, blind endorsement is not restricted to utterances expressible in a given language, or affected by otherwise complicating factors like the occurrence of demonstratives.

As mentioned before, Azzouni constructs a formal language for anaphorically unrestricted quantifications, provides it with a semantics and a proof theory, and establishes its completeness.

4   Critical Engagement

Tracking Reason: Proof, Consequence, and Truth brings into focus controversies involving all three of its topics. Since truth and logical consequence are topics I have thought and written about, it would be fruitful to engage Azzouni on those topics.

4.1 Truth   Here the main contested issue is *deflationism vs. substantivism*. Azzouni is a deflationist about truth, and given his strict distinction between "true" and "truth," his deflationism takes different forms for these subjects. His "true"-deflationism consists in the view that (i) the one and only role of "true" in the vernacular is *blind endorsement*, and (ii) in this role "true" functions as no more than a stand-in for something else, namely, unrestricted anaphora and anaphorically unrestricted quantifiers. Azzouni's *truth*-deflationism consists in the view that nothing (of interest) is common to all truths, and that truth, therefore, is not a substantive attribute of truths.

"*True*."   By a "theory of 'true'" Azzouni understands, in accordance with his functional-analysis approach, "a theory about a piece of language ('true') and its (indispensable) role" (p. 31). So far so good. But the only indispensable role of "true," Azzouni argues, is to partake in *blind endorsements*:

> [T]he role of the truth predicate, in ordinary language, is *not* a genuinely semantic one but serves only to facilitate semantic ascent and descent in blind endorsements. (p. 94, my underline)

And the work it does in blind endorsement is nothing more than a substitute for anaphorically unrestricted pronouns and quantifiers. What this means is that the only (serious) reason we need a truth predicate in our language is as a stand-in for something else, and that if in the course of history our language had acquired certain

technical mechanisms (specifically, a more powerful anaphoric machinery), there would be no need for a truth predicate at all(!).

This sounds like an extremely narrow view of "true"'s role in language, but what Azzouni has in mind is a bit broader: The view is not that "true" is (properly) used only in blind endorsements, but that all its uses can be eliminated by the same (theoretical) mechanisms as those used to eliminate its use in blind endorsement. In particular, its role in stating the truth conditions of sentences can be so eliminated. We can recast

(G)   For all sentences A and B, $(A \vee B)$ is true iff either A is true or B is true,

as

(G$'$)   For all sentences A and B, $(A \vee B)$ iff either A or B.

(Page 27; see also page 36.) Azzouni further acknowledges the possibility of rhetorical uses of "true." But these, he points out, can be ignored.

Still, Azzouni's understanding of "true" (and the theory of "true") is pretty narrow. All the theory of "true" has to do is to show us how to eliminate "true" by introducing certain technical mechanisms into our language. Even the study of reference and satisfaction is outside the boundaries of this theory. Azzouni's notion of "true," therefore, is extremely deflationist.

What Azzouni shows in *Tracking Reason*, however, does not suffice to establish extreme deflationism about "true." First, he does not conduct a comprehensive survey of its uses in the vernacular, nor does he prove that all its possible uses would be (in principle) eliminable by anaphorically unrestricted quantification or any other technical device. All he considers are two uses of "true": its use in blind endorsement and its use in stating the truth conditions of logically-structured statements.

Second, from the fact that some uses of "true" can in principle be eliminated it does not follow that all its uses (actual and potential) can: its being eliminable in some contexts is compatible with its being ineliminable in others. More than that, the fact that truth is (theoretically) redundant in some roles may suggest that these roles are not its main, or defining, roles, that its defining role(s) are other than its eliminable roles. It may very well be the case (to take a line of thought congenial to Azzouni) that it is no more than an accident of evolution that "true" is used in blind endorsement, but its "real" purpose is different.

Moreover, given Azzouni's liberal view of the vernacular—even mathematics is part of the vernacular—theoretical uses of truth count as genuine roles as well. This includes its roles in ethics, epistemology, and other branches of philosophy, where navigating use and mention is not the point. None of these is ruled out by Azzouni.

Finally, even if "true" is eliminable in all its appearances, this does not establish its redundancy. It is possible that by eliminating "true" we obtain grammatical and even meaningful sentences, yet these sentences are significantly impoverished compared to the originals. Take, for example, the statement,

(3)   Some sentence is true although no one can prove it.

As we have seen above, Azzouni rightly says that we can transliterate this language in Anaphorish as

(4)   Some sentence, it, although no one can prove it.

But while (3) has as its subtext the systematic relation between proof-theory and semantics, (4) does not. Indeed, the whole subject of "completeness" will be considerably impoverished under a systematic elimination of alethic terms from our discourse. *It may be an accident that the truth-predicate is used for blind endorsement in our language, but this does not mean it is an accident that we have a truth predicate in our language.*

Truth. Azzouni characterizes a theory of truth as follows:

> A theory of *truth* is a theory, if such is possible, about the systematic uniformities (if any) among *truths*. (p. 31)

And he claims that such a theory is not possible since there cannot be any systematic uniformities among truths:

> [N]othing in general can be said about truths—...there really isn't (really can't be)...anything like a (substantial) theory of truth. (p. 32)

To "give an indication for why I thinks [that]," he continues, "I must bring up considerations argued for [in Azzouni [1]]" (ibid.). Briefly, these considerations are the following:

> I presuppose a metaphysical claim: Nominalism is correct (there are no abstracta). Nevertheless, mathematical statements are *true* and are intertwined with ordinary empirical statements (about things that *do* exist) in such a way that no semantic theory is possible that separates statements that are true (and are solely about things that exist) from statements that are true (and are—at least partially—about fictions). Consequently no general theory of truth—at least of a correspondence sort—is forthcoming. This hardly shows, of course, that no *other* theory of truth is forthcoming—even granting my nominalism; for, no doubt, readers can think of (or anyway, recollect) other ways that truths can be seen as having uniform properties. I'm sceptical, however, that these other options will work because I'm sceptical that, once we desert the correspondence option, there are any genuine uniformities to attribute to the *truths themselves*, as opposed to what may be broadly described as the epistemology of truth: how we establish truths, or supply evidence for them, or something like that. (pp. 32–33)

A lot is packed into the above citations, so let us deal with them in steps:

(i) *A theory of* truth *is a theory, if such is possible, about the systematic uniformities (if any) among* truths.

The view that a theory of truth is a theory of the common features of all truths is widely-spread. But it is a *myth* that such is, or ought to be, a theory of truth. It is a myth that to understand a philosophical subject matter is to understand what is common to all the things that fall under it. And it is a myth that to develop a substantive philosophical theory about X is only, or even mainly, to identify *the common property* of all Xs. (In Sher [7] I called this "the myth of the common denominator.") In philosophy as in science it is important to identify both the systematic uniformities and the systematic differences among the things we are investigating, and finding systematic differences is no less important or informative than finding systematic commonalities. (I have expanded on this and other points made below in Sher [7] and [9].)

(ii)   *I [Azzouni] presuppose a metaphysical claim: Nominalism is correct (there
        are no abstracta).*

This claim, which was argued for in Azzouni [1], is beyond the scope of the present
review. To give Azzouni the benefit of the doubt, I will (a) interpret his argument in
a way that is not dependent on the correctness of nominalism and (b) formulate my
own arguments in a way that is compatible with nominalism.

(iii)   *Nevertheless, mathematical statements are true and are intertwined with ordi-
         nary empirical statements (about things that do exist) in such a way that no
         semantic theory is possible that separates statements that are true (and are
         solely about things that exist) from statements that are true (and are—at least
         partially—about fictions). Consequently no general theory of truth—at least
         of a correspondence sort—is forthcoming.*

The possibility of systematic differences between truths in different areas of
discourse—say, between mathematical and (ordinary) empirical truths—does not
rule out the possibility of a substantive theory of truth. "Truth" is a very broad
philosophical concept, spanning highly divergent subject matters of our thought, and
as such we would expect significant differences between different subject matters,
or clusters thereof, with respect to truth. The existence of significant differences,
however, does not rule out the existence of significant similarities. And therefore
a substantive theory of truth, namely, a theory of the systematic uniformities and
differences among truths, is in principle possible. Furthermore, the existence of
significant differences among truths does not rule out the existence of systematic
uniformities among subsets of truths, so that (contrary to what Azzouni says on page
34) we do not have to deal with such truths on a *case by case* basis. For example,
it is possible that ordinary empirical truths have some systematic uniformities as do
abstract mathematical truths (or abstracta more generally). Studying such uniformi-
ties (and differences) is an integral task of a substantive theory of truth (or a theory
of any diverse and multifaceted subject matter).

This conception of a theory of truth raises the possibility of pluralism with respect
to truth, a view that has recently been proposed by Wright [14] and [13], Lynch [4],
and Sher [9], each offering a different version of pluralism. In my version, a pluralist
theory of truth acknowledges both the substantive uniformities and the substantive
differences among truths, and is bound, like any other theory, by the methodological
desiderata of unity on the one hand and attention to differences on the other. The
plurality is *a plurality of forms of correspondence*, or *a plurality of correspondence
principles of truth*. By regarding all truths as based on forms of *correspondence*
we achieve a great deal of (substantive) unity; by distinguishing different *forms* of
correspondence we take note of systematic differences.

Azzouni criticizes the pluralist approach to truth on the ground that abstract and
empirical statements are so intertwined that we cannot attribute different semantic
principles to them. In a footnote he elaborates,

> a view that separates truths into separate categories according to different the-
> ories of truth that apply to them presumes, to begin with, that truths *can* be
> neatly sorted into various categories, and I've already given reasons to doubt
> this is possible. In Part II of Azzouni [2004], I illustrate in some detail how
> pure mathematics interpenetrates with the empirical (in certain applications of
> physics) in a way that makes it impossible to sort the resulting doctrine into

> "purely" empirical truths and "purely" mathematical ones. This interpene-
> tration phenomena among truths is quite widespread—indeed, it's a form of
> holism. (p. 34)

But Azzouni's point does not apply to pluralism within the bounds of correspon-
dence. It applies to theories that regard some truths as based on correspondence
principles and others on coherence principles. But this is not the kind of pluralism
I am talking about here. Pluralism about correspondence includes pluralism about
reference and satisfaction, and it is perfectly possible that the principles of reference
governing one singular term within a given sentence differ from those governing an-
other singular term within the same sentence. A singular term referring to an abstract
object may connect to it in one way; a singular term referring to a "concrete" object
may connect to it in another way. (I have elaborated this point in Sher [10]).

   But is the correspondence theory of truth—be it pluralist or one-dimensional—
compatible with nominalism? Azzouni thinks it is not. His view, however, is based
on the assumption that there is just one way a correspondence theorist can connect
truth to reality, namely, the way captured by Quine's criterion of ontological com-
mitment. Azzouni himself rejects this criterion, and for a good reason. The mere
fact that "there is" often carries ontological commitments does not mean it always
does. The same idiom can be used in one context in one way, in a different context
in another: convey ontological commitment to, say, physical individuals, but not to
mathematical individuals. The pluralist correspondence theorist, therefore, is as free
to reject Quine's criterion of ontological commitment. As a correspondence theorist
(of my ilk) he claims that truth requires a systematic connection between true sen-
tences (propositions, beliefs,...) and reality, but this connection can take different
forms in different cases. Consider, for example, a simple sentence of the form

(6)   The number of As is α,

where "A" is an ordinary physical predicate and "α" a cardinal expression. Frege
taught us that (6) allows two interpretations, on one of which "α" is a singular term
and on the other of which it is a second-level predicate. We can thus construe (6)
both as

(7) $(\exists x)(x = \text{the number of } A\text{s})$

and as

(8)   (For α-many x) Ax.

Now, the possibility characterized by the following collection of principles and facts
is perfectly compatible with nominalism, assuming Azzouni's conception of natural
language:

   (a) All existing objects are physical.
   (b) All individuals exist.
   (c) Properties are not objects.
   (d) Properties of individuals have both physical and mathematical properties.
   (e) The force of (6) is captured by (8), but due to certain accidental features
        of natural language on the one hand and the human brain on the other, it is
        convenient (fruitful, mandatory) for humans to write (6) as (7).

On the surface (6) commits us to the existence of numbers, but in fact it does not.

(iv) *This hardly shows, of course, that no other theory of truth [than the correspon-
      dence theory] is forthcoming—even granting my nominalism; for, no doubt,
      readers can think of (or anyway, recollect) other ways that truths can be seen
      as having uniform properties. I'm sceptical, however, that these other options
      will work.*

Here I am in agreement with Azzouni.

(v) *I'm sceptical that, once we desert the correspondence option, there are any
     genuine uniformities to attribute to the truths themselves, as opposed to what
     may be broadly described as the epistemology of truth: how we establish
     truths, or supply evidence for them, or something like that.*

Here Azzouni gives expression to the view, which he repeats several times in *Track-
ing Reason*, that there is a sharp boundary between epistemology and the theory of
truth. With this I disagree. Not only is this view unsubstantiated by Azzouni (or any-
one else, to the best of my knowledge), it is incompatible with another view he holds
(and I share): Holism. Holism strongly suggests that epistemology and the theory of
truth are interconnected: that epistemic issues are relevant to the theory of truth, and
issues pertaining to truth are relevant to epistemology. Indeed, if establishing *truths*,
providing evidence for *truths*, and similar activities pertaining to *truth* are central to
epistemology, as Azzouni appears (rightly in my view) to think, then understanding
what it is that we set out to establish, provide evidence for, and otherwise epistemi-
cally relate to, is an important task of a discipline (like philosophy) that takes (these
aspects) of epistemology as its business.

4.2 Logical consequence   The semantic notion of logical consequence, accord-
ing to Azzouni, "is a notion that doesn't actually illuminate our grasp of what fol-
lows from what, but largely encapsulates our ignorance of how inference operates"
(p. 117). It is but a stand-in, or surrogate, for the syntactic notion of logical conse-
quence, *deducibility*.

   Why, then, do we have a semantic notion of logical consequence? Azzouni's
answer is because of the vagaries of language on the one hand and human nature on
the other.

> [T]he vernacular masks... logic—whatever it is—in a very straightforward
> manner: Inference can't turn on the visible syntactic features of the sen-
> tences. Naturally one looks to the subject matter to explain the intuitive force
> of validity..., and eventually one tries to ground this knowledge of necessity,
> that inference seems to carry along with itself, in a mythology of abstracta.
> (p. 186)

But

> our phenomenal grasp of how we reason is utterly misleading. (pp. 186–87)

In fact,

> [s]pecific logical relations between sentences—even if these, phenomenolog-
> ically, have a semantic feel—are recognized by couching them in something
> effective. And this is regardless of the fact that the expressive capacity of our
> language... outstrips anything effective. (p. 146)

However,

> when (psychologically speaking) we haven't access to the rules establishing
> a result, but only to the result itself, we *can't see* how it could have been
> otherwise. (p. 209)

This leads us to think of logical inferences in metaphysical terms:

> [A] proposition or sentence strikes us—intuitively—as unfalsifiable, and then in turn, as something metaphysically binding on the universe around us (so that it's seen not as, at best, an innately endowed, neurophysiologically contingent, limitation on our metaphysical imagination, but rather as a profound logical insight into the metaphysical necessary). (ibid.)

In this way,

> our usage of "consequence" and "validity" as they arise in ordinary reasoning...seem to involve interaction between "true" and modality like so:
>   (1) If S_1 is true, and S_2 is a consequence of S_1, then S_2 *must* be true as well. (pp. 198–99)

Indeed,

> any (ordinary) notion of *understanding*, in the sense that one understands why something follows from something else, seems to clearly require not recourse to the explication of a mechanical process, but recourse to something irreducibly semantic. (p. 187)

But, according to Azzouni, there are serious problems with the semantic account of logical consequence:

A *The semantic account of logical consequence is metaphysical, whereas the ordinary notion of logical consequence is not.*   Azzouni identifies the semantic account of logical consequence with Tarski's model-theoretic definition of this notion:

(LC) A sentence σ is a logical consequence of a set of sentences Γ iff there is no model in which all the sentences in Γ are true and σ is false.[2]

And he interprets Tarski's account as a metaphysical account, of the kind delineated by Etchemendy [2] under the rubric "representational semantics." (Etchemendy himself endorses a different interpretation of Tarski's account, a so-called interpretational one.) On the metaphysical interpretation of Tarski's account models represent metaphysical possibilities: all the metaphysically possible ways the world could have turned out to be. A sentence is logically true, on this construal, iff it is true in every metaphysically possible world; a consequence is logically valid iff it preserves truth in all such worlds.

But this account, Azzouni argues, fails to capture the intended notion of logical consequence:

> [A] metaphysical interpretation isn't part of the intuition of the modality [involved in the ordinary notion of consequence.]...Indeed, carefully looking at (1) [If S_1 is true, and S_2 is a consequence of S_1, then S_2 *must* be true as well], all that seems to be implicitly taken as varying is the truth (values) of sentences. (p. 214)

Metaphysical necessity is the wrong kind of necessity for logical truth and consequence:

> [I]t may be that "$2 + 2 = 4$" is metaphysically necessary, and *can't* be false. No matter. *Metaphysical necessity is not logical necessity*, and so there are possible extensions of the truth predicate without "$2 + 2 = 4$." (ibid.)

That is, as far as logic is concerned, a situation in which a metaphysically necessary sentence like "$2 + 2 = 4$" is false is possible. (Think of standard first-order logic and a regimentation in which "2" and "4" are individual constants.) And the same holds for consequence:

> So too, it may be that there is (metaphysical) necessitation between snow's
> being white, and water's being transparent. *Whatever*: If it isn't a matter of
> *logical* necessity (and it's not), there *can be* extensions of the truth predicate
> containing one of those sentences but not the other. (pp. 214–15)

That is, from the point of view of logic it is possible to vary the distribution of truth
values among sentences so that the first of these sentences turns out true and the
second false. Furthermore,

> a conflation of logical possibility (and necessity) with metaphysical possi-
> bility (and necessity)…leads to *outrageous* amounts of epistemic woe and
> sorrow (i.e., to hopelessly irresolvable questions about how *we*—fallible ani-
> mals with *really poor* imaginations—manage to fix *our* logical principles by
> the invisible *light* of implacably grand metaphysical facts). (p. 216)

To sum up,

> to conceptually connect the Tarskian notion…to the ordinary notion…, it
> seems we must show how model theory is implicitly involved in the ordinary
> concept. But this looks like a hopeless task. (p. 199, parentheses omitted)

Two other ways in which the semantic conception of logical consequence conflicts
with the ordinary conception, according to Azzouni, are the following.

B *The semantic definition of logical consequence fails to capture the intended
modality.* Here Azzouni relies on Etchemendy [2]. While it is an integral part
of the ordinary notion of logical consequence that "the truth of the premises must
guarantee the truth of the conclusion" (ibid., p. 85), the truth of the premises of a
Tarski-consequence, according to Etchemendy, does *not* guarantee the truth of its
conclusion.

C *The ordinary notion of logical consequence is not committed to the mathematical
machinery used by Tarski.*

> [There is a] possibility of asserting consequences of, or the consistency of,
> sentences while denying the existence of the mathematical items crucial to
> Tarski's definitions of these ideas. That such coupled assertions don't seem
> inconsistent suggests Tarski's notions aren't explications of the ordinary no-
> tions but supplantations. (p. 200)

D *Tarski's approach to nonlogical expressions has too much latitude.*

> Tarski's approach has too much latitude: As sentences are taken relative to
> different models, names and predicates may be reinterpreted *arbitrarily* pro-
> vided only that the sentence remains true in each model. This is an extraor-
> dinarily weak constraint which violates the identities of sentences as mean-
> ingful units: In considering various circumstances where Peter jumps, I don't
> want Peterless cases where Jack jumps, or Sam is a sandwich! (ibid.)

E *The semantic relation of logical consequence is not effective, hence, unusable.*
Using "implication" for the semantic relation, Azzouni says,

> implication relations, in general, are quite unusable. On the model-theoretic
> view, for example, it's impossible to see how anyone can determine directly
> (i.e., by checking each such model) that in every model that *A* is true in, *B* is
> true as well. (p. 122)

F *Set-theoretic limitations of the semantic account:*

> [Tarskian] models are sets, and yet there are consistent sets of truths—say, all
> the truths of set theory—that are true of something (all sets) which isn't a set,
> and thus not a model. If such truths are true relative to some model, it must
> be one that "does not have the full set-theoretic reality in its domain (and in
> which '∈' may not even stand for the membership relation). Why on earth
> should anyone believe that there is such a model?". (p. 200; the citation is
> from Field [3]: 31)

Azzouni's proposed solution to these (alleged) problems is, once again, regimentation:

> [One way to] escape whatever obstacles the semantics. . . of the terms of natural languages place in [our way, is to use]. . . regimentation in an artificial language as a touchstone of what we should take to follow from what. (p. 223)

And this way leads, according to Azzouni, to syntactic derivations. Semantic consequences are, therefore, a stand-in for syntactic derivations.

As a corollary, Azzouni argues that the quest for a semantic criterion of logicality—a criterion that distinguishes logical from nonlogical constants—is doomed to failure. Azzouni construes the quest for a criterion of logicality as a quest for a criterion of logicality for natural language:

> The problem [of logical constants] is that there doesn't seem to be a principled way of determining which vocabulary items should be taken to be part of the logical vocabulary of ordinary language, and which items should be assimilated to the nonlogical vocabulary. The issue has bite because what logic—first-order or otherwise—is actually exemplified by a language turns directly on what logical vocabulary it has. (pp. 222–23)

Azzouni recognizes the importance of logical constants for logic:

> [A]ll logical connections between sentences arise only from designated logical vocabulary. (p. 215)

He is also aware that not every constant can serve as a logical constant:

> [T]he topic neutrality of the inference rules. . . prevents the inclusion of specific vocabulary to that of "logic." (pp. 197–98)

Furthermore, he acknowledges that a criterion for logical constants seems to be needed:

> It may seem th[e] project of the codification of our logical principles requires a *principled* distinction between logical and nonlogical vocabulary in the vernacular. (p. 213)

But he believes this requirement cannot be satisfied:

> [M]aking *that* distinction out looks hopeless (ibid.).

Speaking in weaker terms, Azzouni says,

> The normative question of what *should* be included as logical terminology, and what not, is perhaps an irresolvable one—certainly it looks that way with respect to the notion of identity. (p. 197)

And he adds,

> Actually, I. . . deny altogether that the issue [of logical constants] is to be resolved (or *needs* to be resolved) by sorting items of the *vernacular* into logical and nonlogical constants. (p. 198)

There is much that I agree with in Azzouni's approach to logical consequence and logical constants, and much that I disagree with. I agree that logical consequence is not a metaphysical notion and should not be construed as one. I agree that an

adequate account of logical consequence requires regimentation. I agree that logical constants play a central role in logical consequence and that not every constant can serve as a logical constant in an adequate logic. And I agree that the problem of logical constants will not be resolved "by sorting items of the *vernacular* into logical and nonlogical."

But I disagree that the semantic account of logical consequence is not illuminating. I disagree that the semantic notion of logical consequence is a stand-in for the syntactic notion of derivation. I disagree that the reason we have a semantic notion of logical consequence is the accidents of evolution (evolution of natural language on the one hand and human psychology on the other). I disagree that the Tarskian definition of logical consequence fails to capture the intended modality of that notion. I disagree that Tarski's approach allows too much latitude in varying the interpretation (or denotation) of nonlogical expressions. I disagree that the semantic relation of logical consequence is unusable. And I disagree that the project of distinguishing logical and nonlogical constants (in a normative manner) is doomed to failure.

Furthermore, some of Azzouni's complaints about the semantic approach to logical consequence are, in my view, philosophically irrelevant. These include the claims that the ordinary notion of logical consequence is not committed to the mathematical machinery used by Tarski and the claim about the set-theoretic limitations of the semantic account.

Rather than discuss each point separately, let me focus on a few general clusters (For fuller discussions of many of the issues raised below, see Sher [6], [5], [8], and [11].)

A *The need for semantics is not phenomenological but functional, justificatory, normative, explanatory, and theoretical* The motivation for a semantic account of logical consequence is, in my view, functional, justificatory, normative, explanatory, and theoretical, rather than phenomenological or psychological. To construct a theory of logical consequence we begin by asking what is the function of logical consequence and what is needed to fulfill this function? If the function is transmission of *truth*, then the semantic method is, prima facie, a suitable candidate.

Azzouni rightly emphasizes the normative nature of logical rules:

> The rules... are the standards for good and bad inferences. (p. 207)

> [L]ogical principles... function normatively—as standards of good reasoning.
> (p. 208)

But what are the standards for the rules themselves? What makes a rule a good standard for *correct* inference (rather than, say, for short inference, or for eloquent inference, or for funny inference)?

The project of a semantic account of logical consequence is a theoretical project. Azzouni himself emphasizes the theoretical aspect of the metalogical enterprise:

> The point isn't that *particular* instances of reasoning, even patterns of such,
> are obscure to us; that's not so. What's opaque to us is the adequacy—i.e.,
> the global adequacy...—of the reasoning made available to us by a logical
> system. This (global) insight is much harder to grasp, and isn't introspectively
> transparent to *anyone*. (p. 210)

This opacity, Azzouni rightly says, calls for the construction of a theoretical system of derivation. But the same opacity also calls for the construction of a theoretical system of logical consequence, one that supplements the derivational theory. Some

things syntax is good at doing (guiding, illuminating, explaining); others, semantics is good at. Our broad ignorance about the working of logical inference requires both.

"Given the opacity to introspection of our logical principles," Azzouni says, "we should...distrust the significance of intuitions about what is and what's not (conceptually) included in our notions, especially with respect to the ordinary notion of consequence" (ibid.). I agree. But for just this reason we require a *semantic theory* of logical consequence (alongside a syntactic theory): a theory that sorts out, not by introspection, but by functional considerations, what is and what is not a good inference, what is the structure, and justification, of such an inference.

Azzouni criticizes the prevalent emphasis on the "obviousness" of logic, and rightly so:

> [There is] a long tradition of seeing logical principles as *obvious* trivialities—
> this picture of the principles of our reasoning is so *very* seductive: It's the old
> lure of getting something for nothing. (ibid.)

This criticism is an invitation to a theoretical account of those principles, including a semantic account.

"What are the constraints, if any, on the varying of the extension of the truth predicate [in determining logical consequences]?," Azzouni asks. "Well," he answers, "this is...a matter to be empirically established" (p. 215). No. This, in my view, is to a large extent a *normative* matter. It is a normative matter to be figured out, theoretically, based (among other things) on the function of logical consequence.

B *Is the Tarskian approach to logical consequence metaphysical? Is every objectual approach to logical consequence metaphysical?* Somewhat ironically, Azzouni, who criticized Etchemendy and others for considering too few options with regard to our theory of consequence, makes the same oversight. There are just two alternatives to his own analysis of logical consequence, Azzouni tells us, the sociological analysis and the metaphysical analysis. The latter he identifies with the semantic analysis. But the semantic analysis of logical consequence need not be— should not be—metaphysical. Nor is it reasonable to interpret Tarski's semantic analysis of logical consequence as metaphysical.

One mistake that leads philosophers to conflate logical consequence with metaphysical consequence is taking necessity to be the only (major) constraint on an adequate semantic account of logical consequence. This, as I pointed out in Sher [6], was one of Etchemendy's mistakes in interpreting Tarski. And this mistake is repeated by Azzouni. Tarski set *two* major conditions on an adequate notion of logical consequence—necessity and formality. And to arrive at an adequate semantic account of logical consequence (and make good sense of Tarski's definition) we have to treat formality as an objectual as well as a syntactic condition. What the objectual dimension of formality is I will discuss below. But already we can see that mere necessary consequences are not going to turn out logical. Logical consequences are necessary in a particular sense, a formal sense, which is stricter, hence different from, the metaphysical sense. Logical consequence, on this conception, is not metaphysically necessary; it is *formally* necessary. And the notion of formal necessity, being narrower and sharper than that of metaphysical necessity, is spared much of the obscurity and intractability of the latter notion.

The objectual notion of formality, on my proposal, is cashed out in terms of invariance. Specifically, a property (relation) is formal iff it is invariant under all isomorphisms. Given a formal property P and two isomorphic structures, $\langle A, \beta \rangle$ and

$\langle A', \beta' \rangle$, β has the property P in A iff β' has it in A'. Models for logic, on this account, represent formally-possible structures of objects, and since metaphysical properties are not isomorphism-invariant, logical models are significantly different from metaphysical modes (possible-worlds).

The totality of formally-possible structures of objects is delineated by some mathematical theory of formal structure—one or another—and this solves several problems. First, the notion of formal possibility is far more tractable than that of metaphysical possibility. Second, by being able to vary the mathematical theories used in our account of logical consequence we can guard against problems with a particular mathematical realization of the semantic account (e.g., a realization based on ZFC). Thus, to avoid the (purported) problem of proper sets, we can use a class-theory of structures. And to respect nominalist scruples about abstracta we can choose a mathematical theory that yields itself to a nominalistic construal. (I should add that due to the holism and nonfoundationalism that both Azzouni and I subscribe to, no serious problem of circularity or infinite regress is threatening us. On a different note, I should mention that by limiting himself to the example of truth tables as models, Azzouni arrives at an oversimplistic picture of what is involved in constructing an adequate semantics for logic.)

C *Response to criticisms of the semantic account of logical consequence*   I have already responded, or pointed the way to a response, to most of Azzouni's criticisms of the semantic account. Let me briefly address two remaining problems: (i) the modal problem and (ii) the problem of latitude.

(i) *The modal problem*   Models, in logical semantics, represent formally-possible structures of objects, hence truth/preservation-of-truth in all models represents truth/preservation-of-truth in all formally-possible structures of objects.   This means that logical truth/consequence, on the semantic account, is formally-necessary/formally-necessarily-truth-preserving, hence also necessary/necessarily-truth-preserving.

(ii) *The problem of latitude*   The task of logic is to identify formally-necessary consequences, and therefore models need not take into account anything but the formal content of a given sentence. This content does not depend on the content of the nonlogical constants appearing in this sentence; therefore, models need not be constrained by this content. As far as the formal laws (necessities) grounding logical consequence are concerned, whether a given sentence is about Peter jumping or about Jack jumping or about Sam being a sandwich is irrelevant; therefore, the latitude is legitimate. More than that: this latitude plays an important role in the semantic method—the role of registering all formal possibilities with respect to any given sentence.

D *The problem of logical constants*   By identifying the logical with the formally-necessary, and the formal with the isomorphism-invariant, we pave the way to a criterion of logicality based on the invariance condition. This criterion is not intended to "sort items of the *vernacular* into logical and nonlogical constants"; it is intended to guide us in constructing a theoretical system, or a normative theory, of logical consequence. (For a proposal of how to construct a criterion for logical constants based on the isomorphism-invariance criterion for formal properties, see Sher [5] and

references there. How to decide which constants satisfying the logicality criterion to incorporate in our logical system is a question I will defer to another occasion.)

E   *Truth and logical consequence*   There is an important sense in which our theory of logical consequence has to work in the world. It is a fact of life that errors in logic, no less than errors in physics, can cause airplanes to crash, medical treatments to fail, and so on. The notion of truth mediates between logic and the world. This is the reason that logical consequence is, and should be, a relation of preservation (or transmission) of *truth*. It is my view that for logic to work in the world, we need a substantive theory of truth. (For further explanation see Sher [6], [8], [9], and [11].)

5   Conclusion

In spite, and perhaps because, of my differences with Azzouni on several central issues, I found *Tracking Reason: Proof, Consequence, and Truth* a stimulating and absorbing book. It is an original, provocative, and important book, and I strongly recommend it to philosophers, logicians, and mathematicians with interest in truth, logic, language, metaphysics, and the philosophy of mathematics.

Notes

1. In referring to *Tracking Reason* I will only indicate page numbers.

2. Azzouni relates to a contemporary version of Tarski's definition, as I will too. The original definition appears in Tarski [12].

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